From *Trairūpya* via *Hetucakra* to *Uddyotakara*

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Introduction

In this paper we deal with a formal aspect of the logic of the mediaeval Indian Buddhist logician *Dignāga*. We relate his *Hetucakra* to the earlier *Trairūpya* doctrine as well as to the later refinement developed by *Uddyotakara*. The interpretation of *Dignāga*'s and *Uddyotakara*'s findings as refinements of the ancient *Trairūpya* doctrine will be accomplished by a proper permutation of rows and columns of *Dignāga*'s and *Uddyotakara*'s schemes. This way of looking anew at Dignāga's famous "3-by-3-matrix" will provide us with new insights into his classical scheme. Applied to *Uddyotakara*'s results, this will allow us to identify - at first sight - the one problematic case of his otherwise perfect exposition of results.

Thesis and Reason

The standard situation in ancient Indian logic[1] , within the context of the *vāda* (discussion) - tradition, is the following. Someone, let us call him the proponent, presents to his opponent a thesis of the following shape:

Thesis: "There is (or: There occurs) a *dharma* S in a *dharmin* P."

We will not translate the terms *dharma* and *dharmin*. In many contexts, *dharma* denotes a property, and *dharmin* a property-possessor. In European logic, this corresponds roughly to predicate and subject of an Aristotelian proposition, "S is affirmed of (*huparchei*) P",

[1] See M. Tachikawa: "A sixth – Century Manual of Indian Logic (A translation of the *Nyāyapraveśa*). Journal of Indian Philosophy 1, pp. 111 – 145, 1971

where S and P each denotes a *term (horos).*

S stands for the *sādhya,* and P for the *pakṣa.*[2]

Such a thesis, for example, might be "The Self (*ātman)* is eternal." Or, in order to adjust this to the abovementioned structure: "There is eternity (S) in the Self (P)."

Having stated this, the proponent is obliged to give a **reason** for his statement.
Traditionally, such a reason has to be given by the presentation of another *dharma* H, the *hetu*[3], which acts as a sign *(liṅga)*, indicating that the *dharma* S to be proven is indeed present in the *dharmin* P:

Reason: "Because of the *dharma* H."

As an example, let us imagine that the proponent is stating Thesis and Reason as follows:[4]

"This object (*pakṣa*) is manmade (*sādhya*), because it is impermanent (*hetu*)."

After having presented his reason, the proponent is obliged to show that this reason is indeed **valid**, i.e. that it is a reason which in fact proves his thesis.[5]

The *Trairūpya* - doctrine provides three conditions T1, T2, and T3 (see the next section) which have to be checked in order to show that a reason is valid.

The theory of *Trairūpya* , the "three marks of a good reason", is ***the*** sophisticated central tool in ancient Indian logic. The origin of this theory is not known, but there are different early references to it, for example in the Chinese *Shun-Zhong-lun*[6] attributed to Asaṅga

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[2] This terminology is somewhat confusing because, in the context of ancient European logic, the *sādhya* S corresponds to the *predicate* of an Aristotelian proposition, and the *pakṣa* P corresponds to its *subject*; thus S and P have to be interchanged in the European context.

[3] *hetu* corresponds to the Greek *aition* in Aristotle's logic.

[4] This form resembles Aristotle's *enthymeme*, which is a shortened form of the syllogism for rhetorical purposes. It only states Thesis and Reason, omitting explicit mentioning of the major premise. Aristotle's classical example in his *Rhetorics* is as follows: "This man is ill, because he has fever."

[5] Within the framework of the ancient Indian rhetorical scheme of the "five-membered syllogism" (see Randle, H. N., A Note on the Indian Syllogism. *Mind* 23, pp 398 - 414), Thesis and Reason are the first two steps, while steps 3 and 4 deal with the **validity** of the reason. The final step 5 states the result of the argumentation.

[6] Taisho Vol. 30, 1565, p. 42 a

(ca. 395 - 470 A.C.), see Katsura (1985)[7] for a detailed exposition of the development of the *Trairūpya* formula.

While many overlapping topics and methods can be found when contrasting ancient Indian and Greek logic, there seems to be no trace at all of a theory similar to the *Trairūpya* doctrine within European logic.[8]

The three conditions of the Trairūpya doctrine

Let us now return to the fictitious situation of a discussion between proponent and opponent, and let us assume that the proponent has made the statements described above (Thesis and Reason) which lay the basis for a dispute with the opponent on the validity of the reason, and, in consequence, of the thesis.

The Trairūpya doctrine gives the following three conditions to be checked:

T1 *dharma* H, the *hetu*, occurs in *dharmin* P, the pakṣa.

T2 There is a *dharmin* in which S occurs **and** in which H occurs.

T3 There is **no** *dharmin* in which H occurs, and in which S does **not** occur.

By common sense, T3 implies that, whenever H occurs in a *dharmin* P, i.e., if T1 holds, then necessarily S occurs in P – because by T3 it is not possible that H occurs and S does not occur. Thus T1 and T3 together imply the occurrence of S in P, i.e., the validity of the thesis presented by the proponent.

The role of T2 is a kind of "seriousness test" to be delivered by the proponent, ensuring to exclude a blatant nonsensical argumentation: Trying to convince the opponent of the fact that H is a sign for S, the proponent has to adduce *at least one* example where - agreed to by both parties - the two properties H and S (the sign and the property to be proved) occur together.

Let us look at the example mentioned above.

Thesis and Reason: "This object is manmade, because it is impermanent."

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[7] On Trairūpya Formula. In: *Buddhism and its relation to other religion*s, Kyoto 1985

[8] In terms of Aristotelian logic, the first condition of the *Trairūpya* doctrine corresponds to the minor premise of the syllogism, while conditions 2 and 3 together ensure the validity of the major premise.

In this case, the property H="impermanent" and S="manmade" satisfy T2, because everyone has already seen a broken manmade object before (as for instance a pot). In this case, T3 does not hold – it is contradicted by any impermanent thing which is not manmade, for example, lightning. In this case, we might say that, in spite of H being a "serious" sign for S, it is not a *valid* sign.

The discussion between proponent and opponent might be agreed upon to proceed due to the following "protocol"[9]:

0. The proponent states the thesis and the reason.

1. The two parties agree upon **T1** – if not, the thesis has not been proven.

2. Proponent is obliged to give an example according to **T2**. If he fails to give such an example, his thesis has not been proven.

3. Proponent claims **T3**; if the opponent does not agree, he (the opponent) has to procure a counterexample. If he does, the thesis has not been proven, otherwise it has to be accepted.

The *Trairūpya* doctrine, imbedded into these rules of dispute, defines a sophisticated basis for deciding the result of a fair discussion of a controversial thesis[10]. It is interesting to realize the central role of examples in T2 and T3. These examples are *not* an ingredient of any kind of "inductive reasoning", but they serve as suitable and precise means for establishing the validity or invalidity of an argument.

The mediaeval philosopher *Dignāga* developed this method further into a means of strictly proving arguments by rational reasoning. He transformed this tool of the *vāda* (dispute) tradition into an instrument of inference, i.e. for the acquisition and transmission of knowledge.[11] By means of his *Hetucakra,* he laid a firm theoretical basis for his theory.

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[9] I am not aware of any textual basis for this "protocol". Its purpose is to illustrate the power of the *Trairūpya* doctrine for deciding discussions of a certain type as well as the logical function which "simple examples" assume in such a situation.

[10] The protocol of a fair dispute given above is of course only a skeleton. For practical use, it must be supplemented by additional rules; for example, by rules which describe how to proceed in case one of the parties does not accept the example of the other side.

[11] This has been earlier suggested by Eli Franco in "valid reason, true sign" *Wiener Zeitschrift für die Kunde Südasiens*, 34 (1990), pp. 201-202.

The Trairūpya scheme

Let us assume for the following, that the first condition, T1, of the *Trairūpya* conditions has been met, and let us concentrate on the conditions T2 and T3 which concern the relation between *sādhya* S and *hetu* H.

The *Trairūpya* doctrine, depending on these two conditions, may be put into the following table (which is just another representation of the original doctrine):

	T3 not met	T3 met
T2 met	NOT VALID	VALID
T2 not met	NOT VALID	NOT VALID

Table 1.1: *Trairūpya* conditions

The *Trairūpya* doctrine allows us also to specify further two of the "NOT VALID" reasons of Table 1.1.

Let us first consider the case in which T2 as well as T3 are both *not met* (left lower corner).

Because non-occurrence of S means occurrence of non-S, we may replace "S occurs" in T2 by "non-S does not occur", and, in T3, "S does not occur" by "non-S occurs". Thus, we obtain the following equivalent formulations of the two *Trairūpya* conditions:

T2 : There **is** a *dharmin* in which H occurs and non-S does not occur.
T3 : There **is no** *dharmin* in which H occurs and in which non-S occurs.

If these conditions are **not** met, the negated forms of T2 and T3 hold true:

T2': There **is no** *dharmin* in which H occurs and non-S does not occur.
T3': There **is** a *dharmin* in which H occurs and in which non-S occurs.

Comparing T2' and T3' with T2 and T3, we see that

- T2' is identical to T3 after having replaced S by non-S
- T3' is identical with T2 after having replaced S by non-S.

Thus, in case T2 and T3 are both **not met**, the last two *Trairūpya* conditions are met for

proving **non-S**. In this case, H is therefore called a "CONTRADICTORY" reason for S.

There is another observation, concerning the reason in the left upper corner, where T2 is met, and where T3 is not met: In this case, the reason H occurs, by definition of T2 and T3, in a *dharmin* where S holds as well as in *dharmin* where S does not hold. Thus the reason is a sign of neither S nor non-S, and the argument cannot lead to a conclusion – it is inconclusive, or "TOO WIDE".

The invalid fourth case in the lower right corner is called "SPECIAL", because here H does neither occur in a *dharmin* where S occurs, nor in a *dharmin* where non-S occurs.

Thus we are led to the following table which is a slight refinement of the content of the Trairūpya doctrine(12):

	T3 not met	T3 met
T2 met	TOO WIDE	VALID
T2 not met	CONTRADICT.	SPECIAL

Table 1.2: Trairūpya conditions 2 and 3

Dignāga's Hetucakra

In his early work, the *Hetucakraḍamaru*, Dignāga found a refinement of the *Trairūpya* scheme. He split the condition "T2 met" into two mutually exclusive cases:

T2 met
1. H occurs **in all** *dharmin*s in which S occurs
2. H occurs **in some** (not all!) *dharmin*s in which S occurs

He did the same with the condition "T3 not met", thus splitting it into two mutually exclusive conditions

T3 not met
1. H occurs **in all** *dharmin*s in which S does not occur.
2. H occurs **in some** (not all!) *dharmin*s in which S does not occur.

The cases

(12) Speaking formally, we have made use of a symmetry in the formulation of the Trairūpya conditions: The pair (T2,T3), formulated with respect to (H,S), is equivalent to (T3,T2) formulated with respect to (H,nonS).

T2 met H occurs **in no** *dharmin* in which S occurs.

as well as

T3 met H occurs **in no** *dharmin* in which S does not occurs.

remain unchanged.

This new classification is a *subdivision* of the conditions of the *Trairūpya* scheme, thus the scheme of Table 1.2 remains intact - only the first row and the first column of the scheme have to be subdivided into two each, and the 2-by-2-scheme transforms into a 3-by-3-scheme. Therefore, the valid reasons are the two cases in the upper right corner.

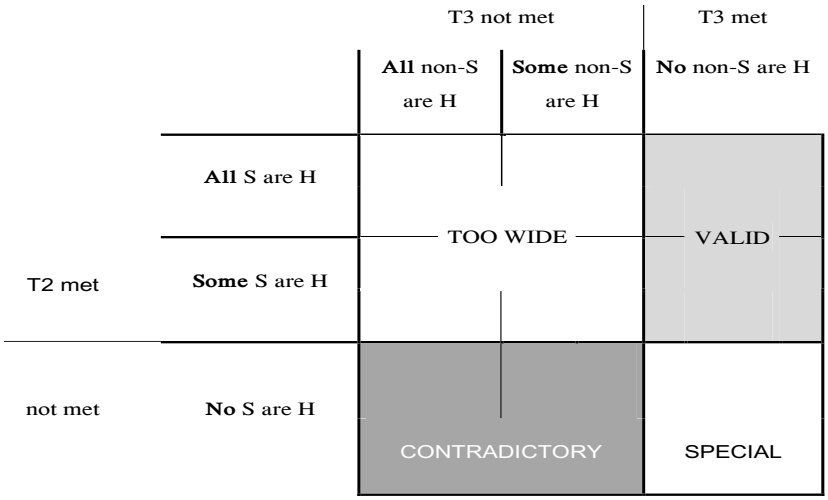

Table 1.3: From *Trairūpya* to *Hetucakra*

Inserting Dignāga's numbering of his nine cases (or, "positions"[13]) of his Hetucakra (from D1 to D9) into this table, we obtain the following scheme:

[13] Richard Hayes, ,,Diṅnāga's views on Reasoning (svārthānumāna)" *Journal of Indian Philosophy* 8 (1980), p.219-277.

		T3			
		ALL	SOME	NO	VALID: D2, D8
	ALL	D1	D3	D2	CONTRAD.: D4, D6
T2	SOME	D7	D9	D8	TOO WIDE: D1, D3, D7, D9
	NO	D4	D6	D5	SPECIAL: D5

Table 1.4: The nine cases of *Hetucakra*

This is in total agreement with Dignāga's "Wheel of Reason", which merely uses a different ordering of rows and columns: interchange the second and third row of the scheme in Table 1.4, and then the second and third column. This makes the table look more like a wheel around the "SPECIAL" position D5, which is now in the center of the matrix:

	T3		
	ALL	NO	SOME
ALL	D1	D2	D3
NO	D4	D5	D6
SOME	D7	D8	D9

Table 1.5: *Dignāga's Hetucakra*

Uddyotakara's extension

Uddyotakara extended the *Hetucakra* by adding seven new cases to Dignāga's scheme[14] [15]. This may be seen as an additional split of the *Trairūpya* scheme, this time affecting the last row and the last column of Table 1.3.

Splitting of the last row refers to the following process:

Consider the "**No-case**" of a *hetu* H (last line of Table 1.3). Out of these cases we select those where there is **no *dharmin* at all**, in which S occurs. This will add a new row to Dignāga's scheme, which is a special case of the original "No"- row. We denote this new row by "Sa=∅" and retain the heading "No" for the remaining cases of a *hetu* which does not occur in a *dharmin* where the *sādhya* S occurs. We then perform the same procedure for the last column of this scheme, thus splitting off the cases in which there is no *dharmin* in which non-S occurs (we denote this by "Vi = ∅") and obtain

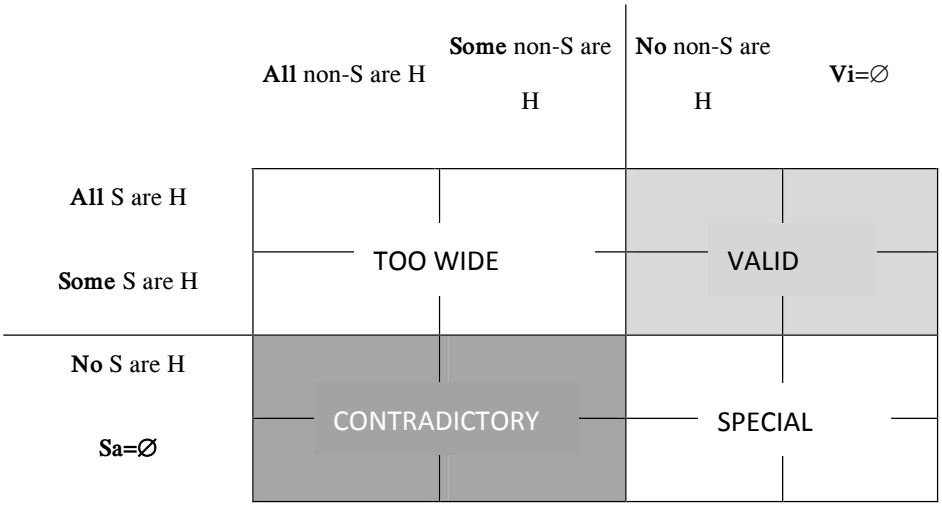

Table 1.6: Final splitting of the *Trairūpya* table

[14] Nyāyavārttika, ed. by V.P. Dvivedin and L.S. Drāvida, Kashi Sanskrit Series, Benares: 1915

[15] Nyāyadarśana with Vātsyāyana's Bhāṣya, Uddhyotakara's Vārttika, Vācaspati Miśra's Tātparyaṭīkā and Viśvanātha's Vṛtti, ed. by Nyaya Tarkatirtha Taranatha and Tarkatirtha Amarendoramohan, Calcutta Sanskrit Series, Calcutta: 1936/44, reprinted Kyoto: Rinsen Book Co. 1982

The result is a table containing 16 cases where – **according to the *Trairūpya* doctrine** – the *hetu-sādhya* pair is of one of the four types:

- **four valid** cases (right upper corner)
- **four contradictory** cases (where H proves non-S)
- **four "too wide"** (thus invalid) cases
- **four "special"** cases in which, according to the *Trairūpya* doctrine, the reason is not valid.

Thus we obtain the following scheme, which is the extension of Table 1.4. We have inserted the "Uddyotakara-positions U1 to U16" according to the order in which he has presented them(16) and, in addition, also Dignāga's position numbers D1 to D9 as of Table 1.4.

According to the Trairūpya doctrine, which is the basis for our classification, there are exactly 4 valid cases (U3, U9, U10, and U11), and the "special cases" U6, U12, U15, and U16 **all represent INVALID cases**.

	ALL	**SOME**	**NO**	**VI=∅**
ALL	D1,U1	D3,U2	D2,U3	U10
SOME	D7,U7	D9,U8	D8,U9	U11
NO	D4,U4	D6,U5	U6	U12
SA=∅	U13	U14	U15	U16

Table 1.7: Classification according to the *Trairūpya* doctrine

Case U15 is of special interest, because *Uddyotakara* claimed it to be a **valid reason**. The method of his proof concerning this case is open to dispute, as it differs completely from the proof of all the other cases(17). But the considerations given in this paper do not imply that

(16) *Uddyotakara* choose a different enumeration for the cases already dealt with by *Dignāga*, and he himself did not present his findings in a "wheel"

(17) See **Yasuhiro Okazaki**, Asādhāraṇa –hetvābhāsa and Uddhyotakara's vyatirekin, Nagoya Studies in Indian Culture and Buddhism: *Saṃbhāṣā* 23, 2003. In this interesting paper, the author develops a formalism which renders valid exactly the 5 positions *including U15) considered to be valid by Uddyotakara himself.

Uddyotakara has made a mistake - they only imply that the *Trairūpya* doctrine cannot be used to prove his claim.

Summary

In this paper we have presented a formal way of showing the connections between three major developments in ancient Indian logic. Beginning by depicting the logical content of the Trairūpya doctrine in the shape of a 2-by-2-matrix, we show how two subsequent refinements of this basic matrix leads to Dignāga's Hetucakra (depicted as a 3-by-3-matrix) and Uddyotakara's theory, displayed as 4-by-4-matrix. This leads us to identify quickly the problematic case of Uddyotakara's classification, namely his position U15 which does not comply to the Trairūpya doctrine.

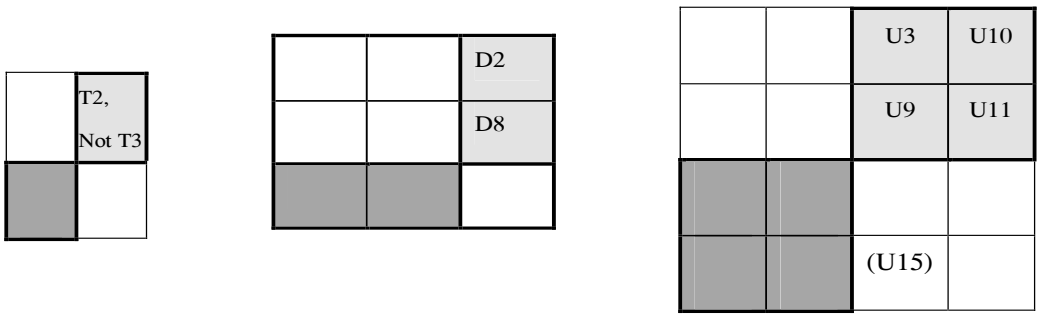

Table 1.8: From *Trairūpya* via *Hetucakra* to *Uddhyotakara*.
Valid positions: light grey; contradictory positions: dark grey;
white positions: other invalid cases.

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< keywords > Hetucakra, Dignāga, Uddyotakara, Trairūpya